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# Jet Noise Predictions Based on Two Different Forms of Lilley's Equation

## Part 2: Acoustic Predictions and Comparison With Data

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## Abstract

The far field acoustic spectra at 90° to the downstream axis of some typical high speed jets are calculated from two different forms of Lilley's equation combined with some recent measurements of the relevant turbulent source function. These measurements, which were limited to a single point in a low Mach number flow, were extended to other conditions with the aid of a highly developed RANS calculation. The results are compared with experimental data over a range of Mach numbers. Both forms of the analogy lead to predictions that are in fair agreement with the experimental data at subsonic Mach numbers. The agreement is not quite as good at supersonic speeds, but the data appears to be slightly contaminated by shock- associated noise in this case.

## I. Introduction

The results of Part 1 can not be used to predict the radiated sound without inputting more specific information about the turbulence structure. In this part we accomplish this objective by using some recent measurements (ref. 1) of the two point fourth order stream-wise velocity correlation spectra that were carried out by Harper-Bourne (ref. 1) at a single point on the centerline of the mixing layer in a low Mach number jet.

## II. The Harper-Bourne Spectrum

Harper-Bourne's results would most closely correspond to

$$H_o\left(\boldsymbol{y},\boldsymbol{\eta},\omega\right)\equiv\frac{1}{\pi}\int_{-\infty}^{\infty}e^{-i\omega\tau}R_{11}^{2}\left(\boldsymbol{y},\boldsymbol{\eta},\tau\right)d\tau \tag{1}$$

with the quasi-normal approximation that is being used in the present analysis.

He divided this quantity into the three components (see his eq. (2.5) and (2.7) on p. 2)

$$H_o = H_o\left(\boldsymbol{y},\mathbf{0},\omega\right) R\left(\boldsymbol{y},\frac{\eta_1}{l_1},\frac{\eta_\perp}{l_\perp},\omega\right) e^{i\omega\tau_p} \tag{2}$$

where $l_1$, $l_\perp$are the spectral stream-wise and transverse length scales (not to be confused with the time domain length scales $L_1$ and $L_\perp$introduced above) and

$$\tau_p \simeq \frac{\eta_1}{U_c} \tag{3}$$

No assumption is made about the decomposition of the correlations into products of their space and time components with this approach.

The first factor can be evaluated from his measurements of $R_{1111}$ ($\boldsymbol{y}$, $\mathbf{0}$, $\tau$), which can be reasonably well represented by the simple exponential $e^{-\lambda|\tau|}$. Taking its Fourier Transform shows that (ref. 20)

$$H_o\left(\boldsymbol{y},\mathbf{0},\omega\right) = \frac{\lambda\widetilde{\rho u_1^4}}{\pi(\lambda^2+\omega^2)} \tag{4}$$

Harper-Bourne was able to obtain a reasonable fit to his data with the non-separable form

$$R = e^{-\sqrt{\overline{\eta}_1^2+\overline{\eta}_\perp^4}}, \tag{5a}$$

$\overline{\eta}_1 \equiv \eta_1/l_1$ and $\overline{\eta}_\perp \equiv \eta_\perp/l_\perp$ by allowing the stream-wise and transverse length scales $l_1$ and $l_\perp$ to be frequency dependant .A better fit might be

$$R = e^{-\sqrt{\overline{\eta}_1^2+\overline{\eta}_\perp^4}}\left(1-\frac{b\overline{\eta}_1^2}{\sqrt{\overline{\eta}_1^2+\overline{\eta}_\perp^4}}\right) \tag{5b}$$

where

$$b = \frac{1+\beta^2}{1+4\beta^2} \tag{6}$$

with $\beta \equiv \pi U_J/U_c$.

Inserting these into equation (2) and using the result in equations (43) and (44) of Part 1 shows that

$$I_\omega\left(\mathbf{x}|\mathbf{y}\right) = C_0^2\,\frac{\lambda\widetilde{\rho u_1^4}\left[\omega^2+\left(\kappa|\nabla U|\right)^2\right]\pi U_c^3\overline{l}_1\overline{l}_\perp^2}{x^2c_\infty^4(\lambda^2+\omega^2)\omega\left[1+\left(2\pi\overline{l}_1\right)^2\right]^{3/2}}\left[1-b\frac{1+4\left(2\pi\overline{l}_1\right)^2}{1+\left(2\pi\overline{l}_1\right)^2}\right], \tag{7}$$

Where the scaled length scales $\overline{l}_1$ and $\overline{l}_\perp$ are defined by

$$\overline{l}_1 \equiv \frac{\omega l_1}{2\pi U_c} \tag{8a}$$

and

$$\overline{l}_\perp \equiv \frac{\omega l_\perp}{2\pi U_c} \tag{8b}$$

Harper-Bourne's figure 13 shows that while $l_1$ and $l_\perp$ are constant at relatively low frequencies it is the scaled length scales $\overline{l}_i$ and $\overline{l}_\perp$ that becoming constant as $\omega \to \infty$. The data is reasonably well represented by the functions

$$\frac{\omega l_1}{2\pi U_J} \approx \frac{1}{2}\left(1 - e^{-2St}\right) \tag{9a}$$

and

$$\frac{\omega l_\perp}{2\pi U_J} \approx 0.15\left(1 - e^{-0.5St}\right) \tag{9b}$$

where

$$St \equiv \omega D \big/ 2\pi U_J \tag{10}$$

As indicated in Part 1, this result was derived only for the first formulation, but it turns out that it will apply to the second as well if $\kappa$ is set to zero and a slightly different formula is used for $C_0^2$. The principle difference between these results is therefore due to the factor $\left[\omega^2 + \left(\kappa |\nabla U|\right)^2\right]$, which does not significantly effect the high frequency behavior of the solution but causes $I_\omega$ to exhibit the dipole-like behavior

$$I_\omega \sim \omega^2 \qquad \text{as } \omega \to 0 \tag{11}$$

in the first formulation and the quadrupole-like behavior

$$I_\omega \sim \omega^4 \qquad \text{as } \omega \to 0 \tag{12}$$

in the second.

## III. Extension of the Harper-Bourne Data

Unfortunately, all of Harper-Bourne's measurements were taken at a single point in a very low Mach number jet, while acoustic predictions require information about the turbulence over the entire noise producing region of the jet. We, therefore attempt to extend his data by using some modeling assumptions along with the Glenn Wind code, which is a RANS code with a standard $k - \varepsilon$ turbulence model. To this end, we first assume that the time scale $\lambda^{-1}$ that appears in equation (53) of Part 1 is proportional to the $k-\varepsilon$ time scale $k/\varepsilon$, i.e., we put

$$\lambda^{-1} \approx C^{\tau}\, k/\varepsilon \tag{13}$$

where $C^{\tau}$ is an adjustable constant. In order to extend equations (9a) and (9b), we assume that the time and velocity scales $D/U_J$ and $U_J$ are proportional to the k – ε time and length scales $k/\varepsilon$ and $k^{1/2}$ respectively to obtain

$$\frac{\omega l_1}{2\pi U_J} \approx C^l k^{1/2}\left[1-e^{-C^S\left(\omega k/2\pi\varepsilon\right)}\right] \tag{14a}$$

and

$$\frac{\omega l_1}{2\pi U_J} \approx 0.3 C^l k^{1/2}\left[1-e^{-0.25 C^S\left(\omega k/2\pi\varepsilon\right)}\right] \tag{14b}$$

where the constants $C^l$ and $C^S$ are determined by requiring that equations (9a) and (9b) be in agreement with Harper-Bourne’s measurements at the Harper-Bourne measuring point and Mach number when $k$ and ε are calculated from the Wind code. A reasonably good approximation is obtained by putting $C^l \approx 3.3$ and $C^S \approx 0.40$. To be consistent with these extensions it is necessary to put

$$\beta = \frac{2\pi C^l k^{1/2}}{U_c} \tag{15}$$

in equation (6).

In the first formulation the constant $C_o$ is related to the ratio $\Gamma r$ (defined in equation (42) of Part 1 by equation (45) of Part 1), which for $\gamma = 1.4$ becomes

$$C_o^2 \approx \frac{0.43}{3}\left(\Gamma r\right)^2 + 0.01 \tag{16a}$$

and in the second by

$$C_o^2 = \frac{1}{4}\left(\Gamma r\right)^2 \tag{16b}$$

Unfortunately, Harper-Bourne only measured the steam-wise and not the transverse velocity correlations so that Γ is essentially unknown. We therefore treat $C^{\tau}$ and $C_o$ as adjustable constants, whose determination is described in the next section. It is necessary to know the square root in equation (46) of Part 1 in order to determine κ, but again, Harper- Bourne does not provide enough data to determine this quantity. We estimate its value to be less than one, however.

## IV. Comparison with acoustic Measurements and Discussion

RANS solutions for Mach 0.50, 0.90, and 1.5 cold jets were obtained from the WIND code with upstream nozzle conditions specified in terms of plenum temperature ratio $T_r$ and the pressure ratio .The predicted turbulent kinetic energy distributions for the three jets are shown in figure 1.

The far-field spectra at $90^o$ to the jet axis were calculated for these jets on the arc R/D = 100 by summing the point result (7) over the noise producing region of the jet. Figures 2 through 4 show the comparison between these results and the subsonic SHJAR data recently acquired at NASA Glenn Research Center and correctly expanded supersonic data obtained at Langley Research Center. Atmospheric attenuation was removed from all measurements in order to make lossless comparisons with the predictions. The agreement is better in the subsonic case, but it is likely that the supersonic data contains a small amount of shock associated noise that is not accounted for by the theory.

The adjustable constants $C^{\tau}$ and $C_o$ were determined by obtaining the best fit with the Mach 0.5 data. The resulting value of $C^{\tau}$ turns out to be 0.10. Figure 3 shows that there is almost no dependence on the parameter $\kappa$ when its value is in the estimated range $0 < \kappa < 1$, which means that these data comparisons were not discriminating enough to distinguish between these two formulations. The hope is that similar comparisons for hot jets or jets with more complex flow fields will provide the required selectivity.

## V. Concluding Remarks

The research was initially motivated by the desire to distinguish between the two forms of the acoustic analogy described above. Unfortunately the results turned out to be inconclusive-with both forms of the analogy yielding reasonable agreement with the data. Our hope is that similar comparisons for hot jets or jets with more complex flow fields will provide the required selectivity. But until this is done, our recommendation would be to base the jet noise predictions on the second formulation, since it leads to much simpler formulas at angles other than 90°.

## References

1. Harper-Bourne, M., “Jet Noise Turbulence Measurements,” AIAA Paper 2003–3214, 2003.
2. Pope, S.B., “Turbulent Flows,” Cambridge University Press, 2000, pp. 70 and 144.
3. Khavaran, A., Bridges, J., and Freund, J.B., “A Parametric Study of Fine-Scale Turbulence Mixing Noise,” NASA/TM—2002-211696, 2002
4. Norum, T., NASA Langley Jet Noise Lab Generic Nozzle Narrow Band Spectra, vol. 1, 1994 (unpublished, available electronically).

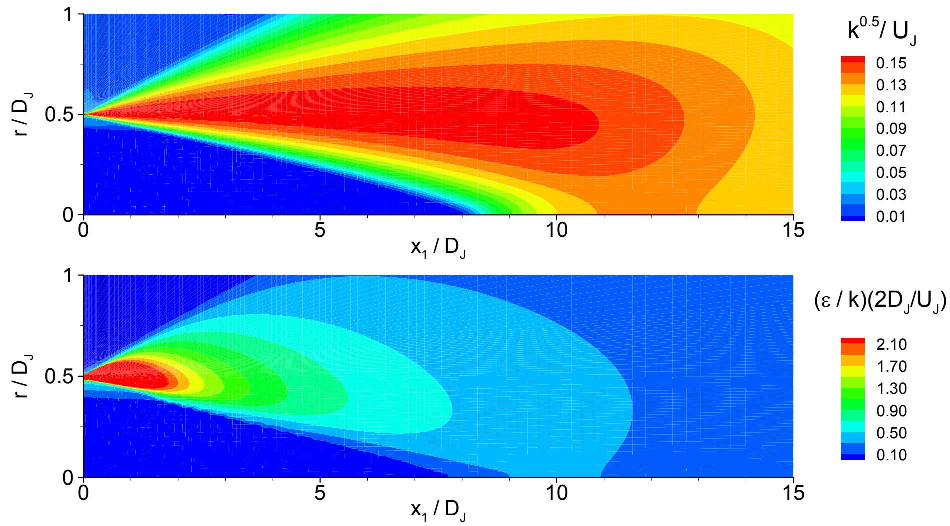


Figure 1(a).—As figure 1 but Predicted turbulent kinetic energy (top), and frequency scale (bottom) for a 2 in. diameter cold jet at Mach 0.50 $r \equiv |\boldsymbol{y}_\perp|$.

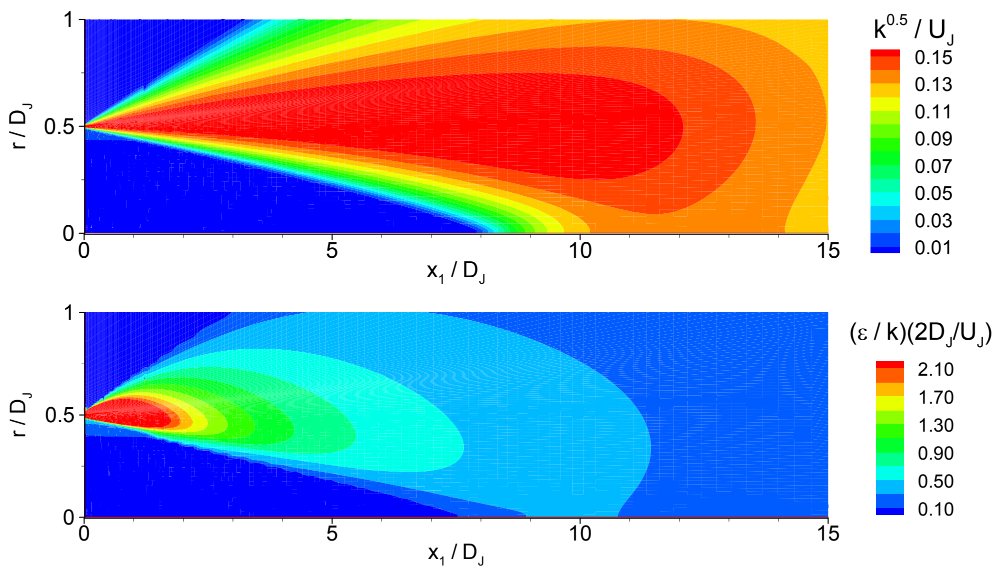


Figure 1(b).—As figure 1 but predicted turbulent kinetic energy (top), and frequency scale (bottom) for a 2 in. diameter cold jet at Mach 0.90 $r \equiv |\boldsymbol{y}_\perp|$.

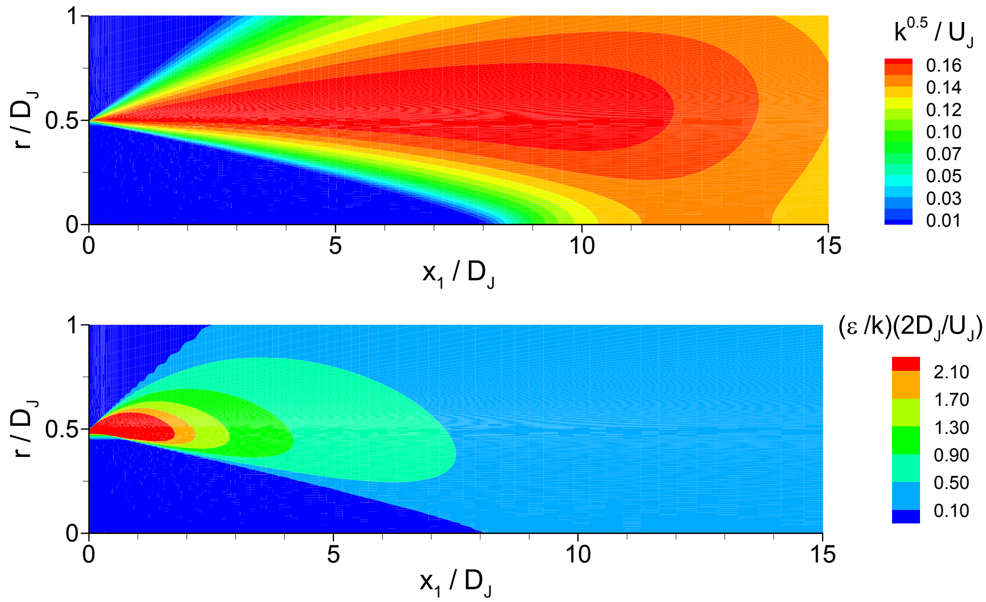


Figure 1(c).—As figure 1 but predicted turbulent kinetic energy (top), and frequency scale (bottom) for a Mach 1.50 convergent-divergent nozzle with 1.68 in. exit diameter $r \equiv |\boldsymbol{y}_\perp|$.

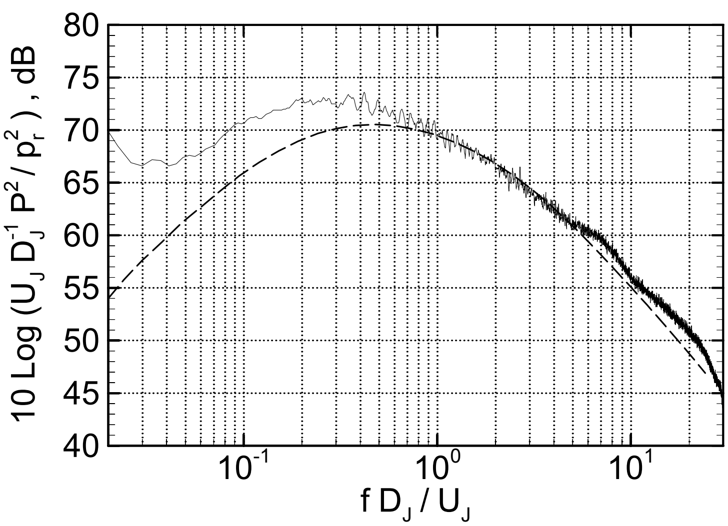


Figure 2.—Spectrum at 90° and at *R/D* = 100 for a Mach 0.50 cold jet. Prediction (dashed line); data (solid line).

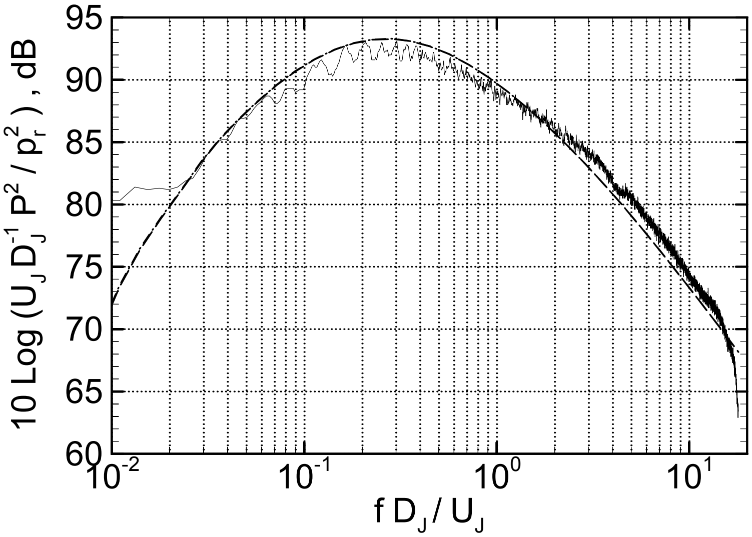


Figure 3.—As figure 1 but for a Mach 0.90 cold jet. Prediction with κ = 0.0 (dashed line); κ = 0.90 (dash-dot); data (solid line).

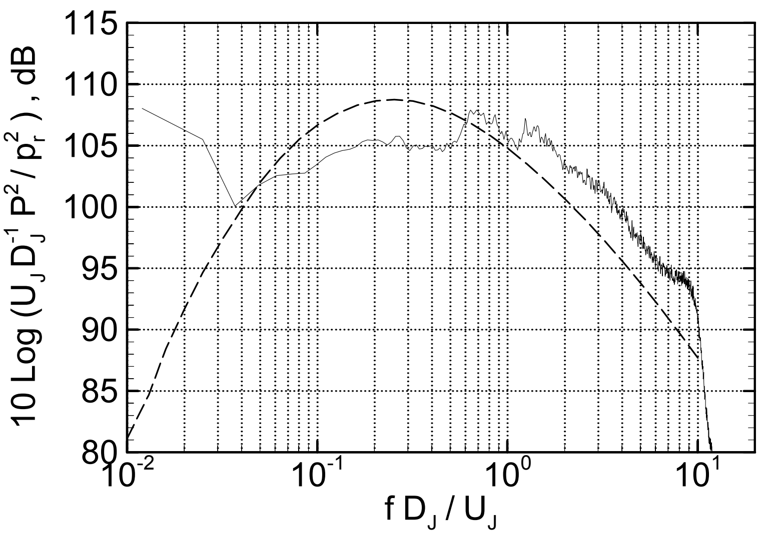


Figure 4.—As figure 2 but for Mach 1.5 cold jet.

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The formulations of the acoustic analogy developed in Part 1 are combined with some recent measurements of the appropriate turbulent source function in order to calculate the far field acoustic spectra at 90° to the downstream axis of some typical high speed jets. The source measurements, which were limited to a single point in a low Mach number flow, were extended to other conditions with the aid of a RANS code with a k -e turbulence model. The results are compared with experimental data over a range of Mach numbers. Both forms of the analogy lead to predictions that are in fair agreement is not quite as good at supersonic speeds, but the data appears to be slightly contaminated by shock-associated noise in this case.

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